Smearing of the 2D Kohn anomaly in a nonquantizing magnetic field: Implications for the interaction effects

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Thermodynamic and transport characteristics of a clean two-dimensional interacting electron gas are shown to be sensitive to the weak perpendicular magnetic field even at temperatures much higher than the cyclotron energy, when the quantum oscillations are completely washed out. We demonstrate this sensitivity for two interaction-related characteristics: electron lifetime and the tunnel density of states. The origin of the sensitivity is traced to the field-induced smearing of the Kohn anomaly; this smearing is the result of curving of the semiclassical electron trajectories in magnetic field.

Introduction. Consider a high-mobility 2D electron gas in a nonquantizing perpendicular magnetic field, B, so that the condition $k_F l \gg 1$, where k_F is the Fermi momentum and $l = (\hbar c/eB)^{1/2}$ is the magnetic length, is met. It is a common knowledge that the thermodynamic and transport characteristics of the electron gas are strongly (in oscillatory manner) sensitive to B at temperatures $T < \hbar\omega_c$, where $\hbar\omega_c = \hbar^2/ml^2$ is the cyclotron quantum, and m is the effective mass. It is also commonly accepted that that the sensitivity to B vanishes rapidly as T exceeds $\hbar\omega_c$. A natural question to ask is whether electron-electron interactions in 2D can change the situation. Previous studies (most recent [1, 2]) suggest that the answer to this question is negative. The magnitude of magneto-oscillations still falls off with T as $\exp(-2\pi^2 T/\omega_c)$. The interaction-induced renormalization of ω_c is singular in temperature, however this singularity vanishes in the limit of a weak disorder.

In the present paper we demonstrate that, due to electron-electron interactions, thermodynamic and transport characteristics of the *clean* 2D gas remain sensitive to B at temperatures much *higher* than $\hbar\omega_c$. More specifically, we show that the electron self-energy, $\Sigma(\omega, B)$, contains a correction which scales with B as (ω/ϵ_0), where ϵ_0 is expressed through the Fermi energy, E_F, as

$$\epsilon_0 = \frac{2E_F}{(k_F l)^{4/3}} = \hbar\omega_c (k_F l)^{2/3}, \qquad (1)$$

Eq. (1) sets the temperature scale $T \sim \epsilon_0 \gg \hbar\omega_c$. Then the corresponding spatial scale, $p_0^{-1} = (k_F l)^{4/3}/k_F$, is much smaller than the Larmour radius, $R_L = k_F l^2$. The easiest way to see the relevance of p_0 is to consider the Friedel oscillations, $V_H(r)$, of electrostatic potential created by a scatterer with a short-range potential, $U_{imp}(\mathbf{r})$, in the presence of electron-electron interaction, $V(\mathbf{r} - \mathbf{r}_1)$. Away from the scatterer, $V_H(r)$ modifies from $V_H(r) \propto \sin(2k_F r)/r^2$ in a zero field to

$$V_H(r) = -\frac{\nu_0 g V(2k_F)}{2\pi r^2} \sin\left[2k_F r - \frac{(p_0 r)^3}{12}\right], \qquad (2)$$

at finite B. Here $\nu_0 = m/\pi\hbar^2$ is the free electron density of states, $V(2k_F)$ is the Fourier component of $V(\mathbf{r})$, and the parameter g is defined as [3] $g = \int U_{imp}(\mathbf{r}) \, d\mathbf{r}$. Eq. (2) is valid within the domain $k_F^{-1} \lesssim r \lesssim R_L$, so that $(p_0 r)^3/12$ in the argument of sine does not exceed the the main term, $2k_F r$. As follows from (1), the momentum p_0 is intermediate between R_L^{-1} and k_F.

The argument of Eq. (2) can be inferred from the simple qualitative consideration. Classical trajectory of an electron in a weak magnetic field is *curved* even at the spatial scales much smaller than R_L. Due to this curving, the electron propagator, $G(\mathbf{r}_1, \mathbf{r}_2)$, between the points $\mathbf{r}_1$ and $\mathbf{r}_2$ contains, in the semiclassical limit, a phase $\hbar^{-1} S(\mathbf{r}_1, \mathbf{r}_2) = k_F \mathcal{L}$, where $\mathcal{L}$ is the length of the arc of a circle with the radius R_L, that connects the points $\mathbf{r}_1$ and $\mathbf{r}_2$, see Fig. 1a. Since the Friedel oscillations are related to the propagation from $\mathbf{r}_1$ to $\mathbf{r}_2$ *and back*, it is important that two arcs, corresponding to the opposite directions of propagation, define a *finite* area, $\mathcal{A}$, so that the product $G(\mathbf{r}_1, \mathbf{r}_2) G(\mathbf{r}_2, \mathbf{r}_1)$ should be multiplied by the Aharonov-Bohm phase factor, $\exp[iB\mathcal{A}/\Phi_0]$. Then the phase, Θ, of this product is equal to

$$\Theta(\mathbf{r}_1, \mathbf{r}_2) = \frac{2}{\hbar} S(\mathbf{r}_1, \mathbf{r}_2) - \frac{B\mathcal{A}(\mathbf{r}_1, \mathbf{r}_2)}{\Phi_0}. \qquad (3)$$

From simple geometrical relations $r = |\mathbf{r}_1 - \mathbf{r}_2| = 2R_L \sin(\delta/2)$, $\mathcal{L} = R_L \delta$ and $\mathcal{A} = 2R_L^2(\delta - \sin\delta)$, we find for $r \ll R_L$ that $\Theta = 2k_F r - (p_0 r)^3/12$, which coincides with the argument in Eq. (2). We emphasize, that the conventional way [4] of incorporating magnetic field into the semiclassical Green's function neglects the curvature of the electron trajectories. This incorporation would not capture the modification (2) of the Friedel oscillations.

In the subsequent sections we trace how the scale, p_0, originates from the smearing of the Kohn anomaly in a magnetic field, and explore the consequences of this smearing for two interaction-related characteristics of the electron gas: electron lifetime and the tunneling density of states in the ballistic regime.
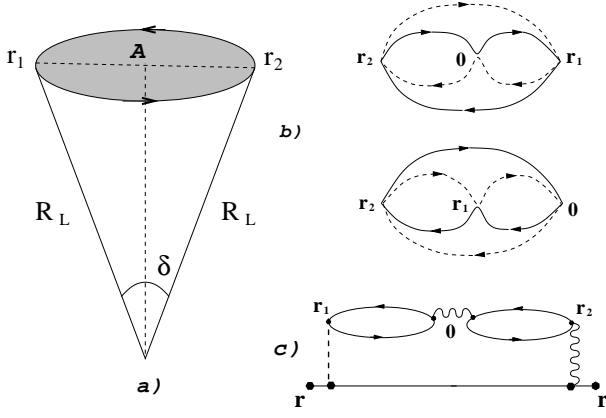
Polarization operator. Curving of the classical elec-

FIG. 1: (a) Origin of the "magnetic" term in the argument Eq. (2): two arcs, corresponding to the opposite directions of propogation, define a finite area, $\mathcal{A}$; (b) Three-scattering processes responsible for magnetic field dependence of the relaxation rate are illustrated schematically; (c) One of three RPA-type diagrams describing the B-dependent correction to the density of states in the ballistic regime: scattering by the impurity occurs at point $\mathbf{r}_1$; two electron-electron scattering acts occur at points $\{0, \mathbf{r}_2\}$.

FIG. 2: (Color online) Derivative, Π', of the polarization operator is plotted from Eq. (4) vs. dimensionless deviation $\delta q/p_0 = (q - 2k_{\rm F})/p_0$ from the Kohn anomaly. Dashed line shows Π' in a zero magnetic field. Inset: Π' is plotted from Eq. (9) at temperatures: $2\pi T = \epsilon_0$ (1), and $2\pi T = 1.5\epsilon_0$ (2).

tron trajectory in a weak magnetic field smears the Kohn anomaly in the polarization operator, $\Pi(q)$. On the quantitative level, it is convenient to describe this smearing for the derivative $\Pi'(q) = d\Pi(q)/dq$. In a zero field and at zero temperature, $\Pi'(q)$ is equal to $-(m/2\pi)\Theta(\delta q)/(\delta q k_{\rm F})^{1/2}$, where $\delta q = (q - 2k_{\rm F})$, and $\Theta(x)$ is the step-function [5]. As shown below, for a non-quantizing field, $\Pi'(q)$ assumes the form

$$\Pi'(q) = -\frac{m}{(k_{\rm F}p_0)^{1/2}} \, Ai\left(\frac{\delta q}{p_0}\right) \, Bi\left(\frac{\delta q}{p_0}\right), \quad (4)$$

where $Ai(z)$ is the Airy function, and $Bi(z)$ is another solution of the Airy equation defined, $e.g.$, in Ref. 6. The rhs of Eq. (4) is plotted in Fig. 2. It is seen that the singularity at $q = 2k_{\rm F}$ is smeared by the magnetic field in a rather peculiar way: for positive $\delta q \gg p_0$ the $(\delta q)^{-1/2}$-behavior is restored. However, for large negative $\delta q/p_0$, the derivative $\Pi'(q)$ approaches zero $with$ $oscillations$, namely, as $\cos[4(|\delta q|/p_0)^{3/2}/3]/(|\delta q|)^{1/2}$. These oscillations have the same physical origin as the term $(p_0 r)^3/12$ in the phase of the Friedel oscillations (2).

$Derivation$ of $Eq.$ (2). We start from the general expression [7] for the polarizability in a magnetic field

$$\Pi(q) = -\frac{2m}{\pi} \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \frac{(-1)^{(n_2-n_1)}\left(f_{n_1} - f_{n_2}\right)}{n_2 - n_1}$$
$$\times \exp(-q^2l^2/2) \, L_{n_1}^{n_2-n_1}\left(\frac{q^2l^2}{2}\right) L_{n_2}^{n_1-n_2}\left(\frac{q^2l^2}{2}\right), \quad (5)$$

where $L_{n_1}^{n_2-n_1}(x)$ and $L_{n_2}^{n_1-n_2}(x)$ are the Laguerre polynomials, and $f_n = \{\exp[(n - N_{\rm F})\hbar\omega_c/T] + 1\}^{-1}$, with
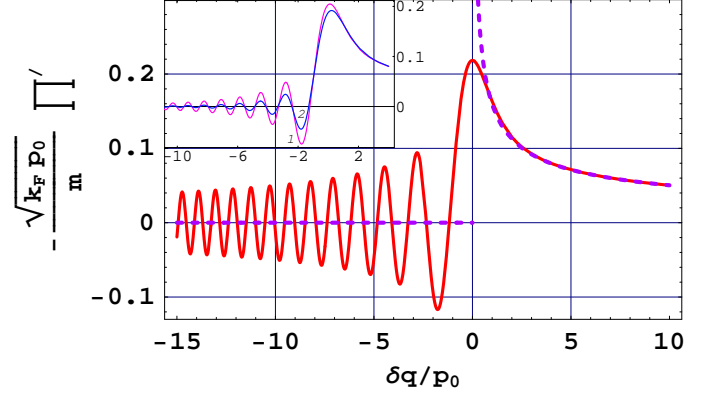
$N_{\rm F} = E_{\rm F}/\hbar\omega_c$, is the Fermi distribution. At small $q \ll k_{\rm F}$ Eq. (5) yields [7] $\Pi(q) = -(m/\pi)[1 - J_0^2(qR_{\rm L})]$, i.e., the characteristic scale is $q \sim R_{\rm L}^{-1}$. For $(q - 2k_{\rm F}) \ll k_{\rm F}$ it is convenient to perform the summation over the Landau levels with the help of the following integral representation of the Laguerre polynomial

$$L_m^n(x) = \frac{1}{2\pi} \int_0^{2\pi} \frac{d\theta}{(1 - e^{i\theta})^{n+1}} \exp\left\{\frac{xe^{i\theta}}{e^{i\theta} - 1} - im\theta\right\} \quad (6)$$

In the vicinity $q = 2k_{\rm F}$ Eq. (6) contains a small factor $\exp(-q^2l^2/2)$. This factor is compensated by the product of the Laguerre polynomials, since each of them is $\propto \exp(x/2)$, which comes from the exponent in Eq. (6) taken at $\theta = \pi$. With contribution from the vicinity of $\theta \sim \pi$ dominating the integral (6), we can expand the integrand around this point as $\exp[x/2 + i\pi m + i\phi(\psi)]/2^{n+1}$, where $\psi = (\theta - \pi)$, and the phase, $\phi(\psi)$, is

$$\phi(\psi) = \left(\frac{x}{4} - m - \frac{n+1}{2}\right)\psi + \frac{x\psi^3}{48}. \quad (7)$$

Now we make use of the fact that only relatively small number $\sim (k_{\rm F}l)^{2/3} \ll N_{\rm F}$ of Landau levels around $E_{\rm F}$ contribute to the sum Eq. (6). This suggests that we can present n_1 and n_2 as $n_1 = N_{\rm F} + m_1$ and $n_2 = N_{\rm F} - m_2$, respectively, and extend the sum over m_1, m_2 from $-\infty$ to $+\infty$. After that, the summation over Landau levels can be easily carried out with the help of the identity

$$\sum_{m_1,m_2=-\infty}^{\infty} \frac{f_{N_{\rm F}-m_1} - f_{N_{\rm F}+m_2}}{m_1 + m_2} \cos\left[(m_1 - m_2)\alpha + \beta\right]$$
$$= \frac{2\pi^2 T \cos\beta}{\hbar\omega_c \sinh\left(2\pi|\alpha|T/\hbar\omega_c\right)}. \quad (8)$$

Upon substituting Eq. (6) into Eq. (5) and using Eq. (8),

one of the angular integrations can be performed:

$$\Pi'(q,T) = -\frac{mT}{2^{1/6}(\pi k_F p_0)^{1/2}\epsilon_0} \int_0^\infty \frac{dx\, x^{1/2}}{\sinh(2\pi xT/\epsilon_0)}$$
$$\times\ \sin\left(2^{2/3}\frac{\delta q}{p_0}\, x + \frac{1}{3}x^3 + \frac{\pi}{4}\right) \qquad (9)$$

It can be shown that in the limit $T \to 0$ the integral (9) reduces to the product $Ai\,(\delta q/p_0)\cdot Bi\,(\delta q/p_0)$, which falls off as $1/(\delta|q|)^{1/2}$ and oscillates, see Fig. 2. As the difference $2k_F - q$ increases and becomes comparable to k_F, these oscillations cross over to the "classical" oscillations $\Pi'(q) \propto J_0(qR_L)J_1(qR_L) \propto \cos(2qR_L)$.

The behavior of $\Pi'(q,T)$ at finite temperatures is shown in Fig. 2. It is seen that increasing T suppresses the oscillatory behavior of $\Pi'(q,T)$ starting from $2\pi T \approx 1.5\epsilon_0$. Now the large-distance behavior of the potential, created by the short-range impurity, can be expressed directly through $\Pi'(2k_F + Q)$ as follows

$$V_H(r) = \frac{V(2k_F)g}{2(\pi k_F r)^{3/2}} \int_{-\infty}^\infty dQ \sin\left[(2k_F + Q)r - \frac{\pi}{4}\right] \Pi'(Q,T).$$

In the limit $T \to 0$ we immediately recover Eq. (2).

Note that, in 3D, the Friedel oscillations in magnetic field were addressed in a number of papers spanning almost four decades [8, 9, 10, 11]. However, the authors of Refs. 8, 9, 10, 11 were unable to carry out the summation over Landau levels in the 3D version of Eq. (5).

The above derivation of the polarization operator near $q = 2k_F$ can be easily extended to finite frequencies. For simplicity we present only a zero-temperature result

$$\Pi'(\omega,q) = \frac{m}{2(k_F p_0)^{1/2}} \sum_{s=+,-} \left[-Ai\left(\frac{\delta q_s}{p_0}\right) Bi\left(\frac{\delta q_s}{p_0}\right) \right.$$
$$\left. + \frac{2\omega}{v_F p_0} Ai\left(\frac{\delta q_s}{p_0}\right) Ai'\left(\frac{\delta q_s}{p_0}\right) \right], \qquad (10)$$

where $\delta q_\pm$ are defined as $\delta q_\pm = (\delta q \pm \omega/v_F)$. It

is straightforward to check that in the limit $l \to \infty$ Eq. (10) reproduces the zero-field result [5] $\Pi'(q,\omega) \propto [(\delta q_+)^{-1/2} + (\delta q_-)^{-1/2}]$.

Implications. The two most prominent characteristics of a clean electron gas, that are governed by the Kohn anomaly, are the energy dependence of the electron lifetime, τ_e^{-1}, and the nonanalytic correction, $\delta C(T) = \tilde\gamma T^2$, to the Fermi-liquid expression, $C(T) = \gamma T$, for the specific heat. Both quantities are being extensively studied [12, 13, 14, 15, 16, 17, 18, 19]. Below we demonstrate that, in a weak magnetic field, τ_e^{-1} acquires a correction, which is singular in B. The standard expression [17, 18] $\tau_e^{-1}(\omega) = (1 + f_s^2)(\omega^2/4\pi E_F) \ln(E_F/\omega)$ is derived within the random-phase approximation (RPA). Here $f_s = r_s/(r_s + \sqrt{2})$, and r_s is the interaction parameter. The RPA definition of τ_e^{-1} can be conveniently cast in the form

$$\frac{1}{\tau_e(\omega)} = \frac{4}{\nu_0} \int_0^\omega d\Omega \int \frac{d^2q\, d^2p}{(2\pi)^4} A(\Omega,\mathbf{p})A(\omega,|\mathbf{q}+\mathbf{p}|)$$
$$\times\ \mathrm{Im}\left[\frac{1}{\Pi(\omega-\Omega,\mathbf{q}) - q\nu_0/(\sqrt{2}k_F r_s)}\right], \qquad (11)$$

where $A(\omega,\mathbf{q})$ is the spectral function related to the retarded, G^R, and advanced, G^A, Green functions in a standard way: $A = (G^R - G^A)/2\pi i$. In a zero magnetic field, the product of the spectral functions in Eq. (11) accounts for the momentum and energy conservation. The B-dependence of τ_e^{-1} comes from the momenta close to $2k_F$ in (11). Hence, to extract this dependence, one should expand the polarization operator in the denominator of Eq. (11) in the vicinity of $2k_F$, and employ Eq. (10). However, the first order expansion *does not* result in the B-dependence of τ_e^{-1}. This happens due to the cancellation of the B-dependent contributions coming from the polarization operator and from the product of the spectral functions. Because of this cancellation, the B-dependence of τ_e^{-1} emerges upon expansion of the denominator in (11) up to the *second* order in $\tilde\Pi(q,\omega) = \{\Pi(q,\omega) - \Pi(2k_F,\omega)\}$ which amounts to replacement $[\Pi(q,\omega) - q\nu_0/\sqrt{2}k_F r_s]^{-1} \to -\tilde\Pi^2(q,\omega)f_s^3/\nu_0^3$. It is much more convenient to present the result of this expansion in the coordinate rather than in the momentum space, where B-dependent contribution to the relaxation rate assumes the following form

$$\left\{\tau_e^{(2)}(\omega,B)\right\}^{-1} = -\frac{4f_s^3}{\nu_0^4} \int_0^\omega d\Omega \int d\mathbf{r_1} d\mathbf{r_2} A(\Omega,|\mathbf{r_1}-\mathbf{r_2}|)A(\omega,|\mathbf{r_1}-\mathbf{r_2}|) \,\mathrm{Im}\left[\tilde\Pi(\omega-\Omega,\mathbf{r_1})\tilde\Pi(\omega-\Omega,\mathbf{r_2})\right], \qquad (12)$$

where $\quad A(\omega,\mathbf{r}) = \frac{\nu_0}{(2\pi k_F r)^{1/2}} \sin\left[k_F r + \frac{\pi}{4} - \frac{(p_0 r)^3}{24} + \frac{\omega r}{v_F}\right] \quad$ and $\quad \tilde\Pi(\Omega,\mathbf{r}) = -\frac{\nu_0}{2\pi r^2} \sin\left[2k_F r - \frac{(p_0 r)^3}{12}\right] \exp\left\{i\frac{\Omega r}{v_F}\right\}$

are the spectral function in the coordinate space, and the

inverse Fourier transform of Eq. (10), respectively. The

major contribution to the integral (12) comes from the domain in which all three points $\{0, r_1, r_2\}$ are close to the straight line (see Fig. 1b), so that the rapid oscillations in the phase factors cancel each other. At $B = 0$, this restriction causes a relative smallness, by a factor $\sim (\omega/E_F)^{1/2}$, of the higher-order correction [16] to the lifetime. At finite B, the integrand in Eq. (12) contains the exponent $\exp\{i[\Theta(|\mathbf{r}_1 - \mathbf{r}_2|) - \Theta(r_1) - \Theta(r_2)]\} = \exp[ip_0^3 r_1 r_2(r_1 + r_2)/4]$; it is this exponent that is responsible for the B-dependence of τ_e^{-1}. Essentially, at $\omega \lesssim \epsilon_0$, the small factor, $(\omega/E_F)^{1/2}$, caused by the angular restriction, is replaced by $(\epsilon_0/E_F)^{1/2} = (2\omega_c/E_F)^{1/3}$. The final result for the correction to the lifetime can be presented as

$$\frac{\delta\tau_e(\omega, B)}{\tau_e(\omega, 0)} = \left(\frac{2f_s^3}{1 + f_s^2}\right) \frac{F(\omega/\epsilon_0)}{\ln(E_F/\omega)} \left(\frac{2\omega_c}{E_F}\right)^{1/3}, \quad (13)$$

where $F(x)$ is the dimensionless function with the magnitude and scale ~ 1. Note, that the correction (13) is strongly (as $B^{1/3}$) singular in B. The origin of this singularity is the cubic dependence of the phase Θ, Eq. (3), on the distance.

The scale, ϵ_0, also manifests itself in the zero-bias anomaly in the tunnel density of states, $\nu(\omega)$, which is closely related to the lifetime (see, *e.g.*, Ref. 20). In the ballistic regime $\omega \gg \tau^{-1}$, where τ is the elastic scattering time [3], only the single-impurity scattering processes determine the ω-dependence of ν. The diagram, responsible for the B-dependence of the *local* density of states (at point $\mathbf{r}$) in the ballistic regime, is shown in Fig. 1c. It describes one impurity scattering at point $\mathbf{r}_1$ and two electron-electron scatterings at points $\{0, \mathbf{r}_2\}$, and thus is quite similar to the process in Fig. 1b. The difference in the analytic expressions for $(\tau_e^{(2)})^{-1}$ and for the *average* RPA $\delta\nu(\omega, B)$ is an extra integration over $\mathbf{r}$, i.e.,

$$\delta\nu(\omega, B) = \text{Im} \frac{6if_s^2}{\pi^2\nu_0^3\tau} \int d\mathbf{r}\, d\mathbf{r}_1 d\mathbf{r}_2\, G_\omega(\mathbf{r}, \mathbf{r}_1) \quad (14)$$
$$\times G_\omega(\mathbf{r}_1, \mathbf{r}_2)\tilde{\Pi}(0, \mathbf{r}_1)\tilde{\Pi}(0, \mathbf{r}_2)G_\omega(\mathbf{r}_2, \mathbf{r}).$$

The factor 3 in (14) reflects the fact that the impurity scattering can occur not only at point $\mathbf{r}_1$ (as in Fig. 1c), but also at points 0 and $\mathbf{r}_2$. Analysis of Eq. (14) yields

$$\frac{\delta\nu(\omega, B) - \delta\nu(\omega, 0)}{\nu_0} = \frac{f_s^2\, F_1(\omega/\epsilon_0)}{E_F\tau} \left(\frac{2\omega_c}{E_F}\right)^{1/3}, \quad (15)$$

where the function $F_1(x)$ is another dimensionless function with scale and magnitude ~ 1. Compared to the $B = 0$ exchange correction [3], $\delta\nu(\omega, 0) = -\nu_0 \ln^2(E_F/\omega)/8\pi E_F\tau$, Eq. (15) contains a small but singular in B factor $(2\omega_c/E_F)^{1/3}$, which has the same origin as (13).

Conclusion. A slight curving of the classical trajectories in a weak magnetic field (as opposed to the drift of the Larmour circle [21, 22]), which had been routinely disregarded since Ref. [4], gives rise to the singular corrections (13), (15) to the lifetime and tunnel density of states, respectively. Although these corrections are parametrically small in semiclassical parameter $(k_F R_L)^{-1} \ll 1$, numerically, they turn out to be sizeable. For example, for $B = 0.2$ T and $n = 2 \cdot 10^{11}$ cm^{-2}, the parameter $(\epsilon_0/E_F)^{1/2}$ is ≈ 0.5. The scale, ϵ_0, translates into a characteristic temperature $T \sim \epsilon_0$, at which the lifetime is sensitive to B. Since $\epsilon_0 \gg \omega_c$, the effects of discreteness of Landau levels $\propto \exp\{-2\pi^2 T/\omega_c\}$ are negligible at this T. Note, that even at $T \sim \epsilon_0$, the inelastic length, $v_F\tau_e(T)$, is $\sim v_F E_F/\epsilon_0^2 \ln(E_F/\epsilon_0)$, i.e., it is bigger than our characteristic spatial scale, p_0^{-1}, in parameter $E_F/\epsilon_0 \ln(E_F/\epsilon_0)$.

Concerning experimental observability of the our predictions, we note that τ_e at $B = 0$ was extracted with high accuracy from the tunneling experiment [23], and was shown to be consistent with RPA calculations [17, 18], (see also [19]). We predict that sensitivity of the width of the Lorentzian in the tunneling dI/dV vs. V characteristics to the magnetic field persists down to low B.

Within the overall picture of 2D electron gas with disorder, the energies $\omega < 1/\tau$ correspond to the diffusive regime, while energies $\omega > 1/\tau$ - to the ballistic regime. Our main results Eqs. (13), (15) apply, when the new energy scale, ϵ_0, belongs to the ballistic domain, i.e., $\epsilon_0 \gg 1/\tau$. This quantifies our assumption that the electron gas is clean. Under the same conditions, it can be "dirty", in the sense, that the mean free path can be much smaller than the Larmour radius, i.e., $\omega_c\tau \ll 1$. Numerical estimate shows that, *e.g.*, for $B = 0.2$ T the condition $\epsilon_0 \gg 1/\tau$ is met even for low mobilities $\sim 10^4$ cm^2/V s.

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